

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures

--	--	--	--	--	--	--	--

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 1

(8 marks)

(a) Solve the following equations for x .

(i) $e^x = 7$.

(1 mark)

(ii) $\log_2(x - 7) + \log_2(x + 7) = 5$.

(3 marks)

(b) Function f is defined by $f(x) = \log_e(x + 5) - 2$. Determine

(i) the equation of the asymptote of the graph of $y = f(x)$.

(1 mark)

(ii) the coordinates of the point on the graph of $y = f(x)$ that has a slope of $\frac{1}{2}$.

(3 marks)

Question 2

(7 marks)

(a) Determine $\frac{dy}{dx}$ when

(i) $y = e^{\sin(x+4)}$.

(2 marks)

(ii) $y = \int_2^x \ln(t^2 - 3t) dt$.

(1 mark)

(b) Determine $\frac{d}{dx}(x \ln(3x))$.

(2 marks)

(c) Hence, or otherwise, determine $\int (\ln(3x) + 5) dx$.

(2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3

(7 marks)

The random variable T is the time in hours that a plumber takes to replace a hand basin in a bathroom and is uniformly distributed between 1 and 3.5 hours. The mean and standard deviation of T are 2.25 and 0.7 hours respectively.

- (a) Sketch the probability density function of T on the axes below. (2 marks)



- (b) Determine

(i) $P(T \geq 3)$. (1 mark)

(ii) the value of the constant k when $P(T > 2.5 \mid T < k) = 0.2$. (2 marks)

The random variable C is the amount in dollars that the plumber charges for labour to replace a hand basin in a bathroom. The plumber charges \$60 per hour plus a fixed call-out fee of \$55.

- (c) Determine the mean and standard deviation of C . (2 marks)

Question 4

(8 marks)

A tank initially contains 23 L of water. Let $V(t)$ be the volume, in litres, of water in the tank t seconds after it is ruptured, so that

$$V'(t) = -\frac{8t}{t^2 + 3} \quad 0 \leq t \leq 30.$$

Determine

(a) $V'(2)$. (1 mark)

(b) $V''(2)$. (3 marks)

(c) $V(2)$. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 5

(7 marks)

A hydraulic press exerts a force of F kilonewtons during the 6 seconds it takes to compress a pellet. Initially it exerts no force and t seconds after it is started, the rate of change of force is given by

$$\frac{dF}{dt} = 3\pi \sin\left(\frac{\pi t}{5}\right).$$

- (a) Use calculus to show that the force exerted by the press is increasing at the greatest rate 2.5 seconds after it starts. (3 marks)

- (b) Determine an expression for the force exerted by the hydraulic press at time t . (2 marks)

- (c) Determine the maximum force exerted by the hydraulic press during the 6 seconds that it operates. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 6

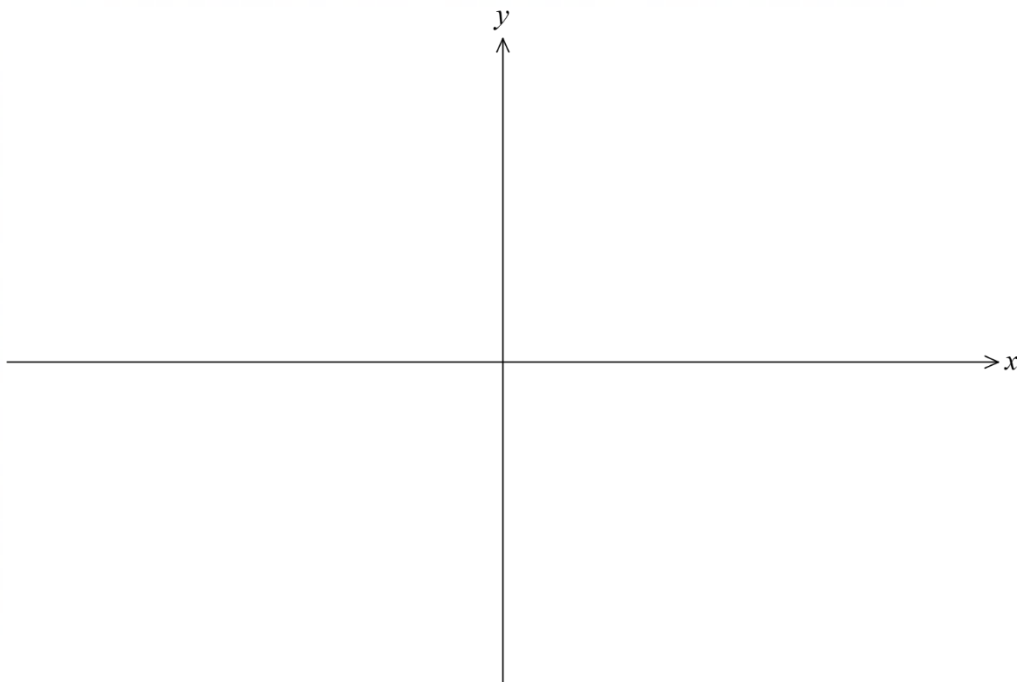
(8 marks)

Let $f(x) = \frac{3 - x^2}{e^x}$, so that $f'(x) = \frac{(x + 1)(x - 3)}{e^x}$ and $f''(x) = \frac{1 + 4x - x^2}{e^x}$.

- (a) Determine the nature and location of all stationary points of $f(x)$. (3 marks)

- (b) Sketch the graph of $y = f(x)$. (5 marks)

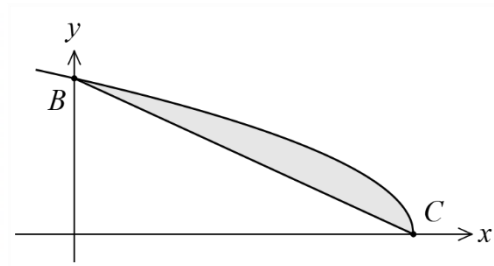
*Some approximations that you may assume are $e \approx 2.7$, $e^2 \approx 7.4$, $e^3 \approx 20$,
 $f(x) \geq 0$ only when $-1.7 \leq x \leq 1.7$ and $f''(x) \geq 0$ only when $-0.2 \leq x \leq 4.2$.*



Question 7

(7 marks)

The graph of the curve $y = \sqrt{4 - x}$ is shown to the right together with the chord BC that joins the points of intersection of the curve with the axes.



(a) Determine the slope of the curve at B .

(2 marks)

(b) Determine the area of the shaded region.

(5 marks)

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

